Multiplication and Division

Children should develop understanding of multiplication as
- repeated addition
- describing an array
- scaling

And an understanding of division as:
- grouping
- sharing

Children can develop this understanding and perform calculations through recording in a variety of ways:
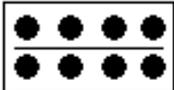- drawing pictures and making marks
- drawing and partitioning arrays
- drawing jumps on number lines
- writing number sentences and using informal and formal written methods
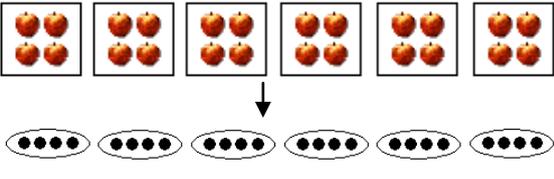
The first table below gives an overview of the calculation expectations for each year group. Statements highlighted in **bold** can be matched to the National Curriculum 2014 programmes of study or non statutory guidance. Other items are suggested approaches for schools to follow to support children's understanding of calculation methods.
The second table in this section sets out how children's recording of calculations might look depending on the mental strategy or written method being used.

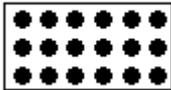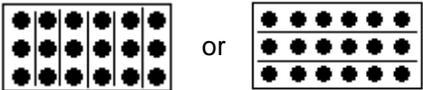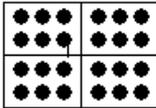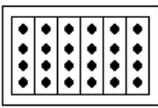| Year	Multiplication and division
1	Children in Year 1 should: • **Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.** • **Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of numbers and quantities** • **Pupils make connections between arrays, number patterns and counting in twos, fives and tens.**
2	Children in Year 2 should: • **Use materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. Begin to relate ideas to fractions and measures** • **Recall and use the multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.** • **Make connections between these tables and connect the 10 multiplication table to place value and the five table to divisions on a clock face** • Use number sentences to show multiplication as repeated addition. • Record multiplications and divisions as jumps on number lines. • **Calculate mathematical statements for multiplication and division within the multiplication tables and use x, ÷ and = signs.** • **Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.** • **Use commutativity and inverse relationships to develop multiplicative reasoning e.g. 4 x 5 = 20 and 20 ÷ 5 = 4** • **Solve problems, including problems in contexts, involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts.**
3	Children in Year 3 should: • Draw pictures and arrays to represent multiplications and divisions if necessary to support understanding, including for situations involving remainders. • Use number sentences and / or number lines to explain multiplication / division as repeated addition / subtraction • Partition arrays to find related number facts for single digit tables facts eg 8 x 4 = (4 + 4) x 4 or 8 x 4 = (5 + 3) x 4. • **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Connect 2, 4 and 8 tables through doubling.** • **Develop efficient mental methods e.g. using commutativity and associativity and using known facts to derive other related facts.** • Use partitioning and use of the distributive law to introduce multiplication and division of a two digit by one digit number. Support this work with images and materials such as arrays and place value counters. • **Write and calculate mathematical statements for multiplication and division using the multiplication statements that they know, including for two-digit numbers times one digit numbers, using mental and progressing to formal written methods of short multiplication and division.** • Use tables facts to **solve problems including missing number, integer scaling and correspondence problems in which n objects are connected to m objects**

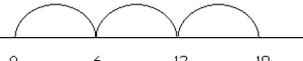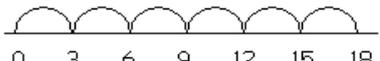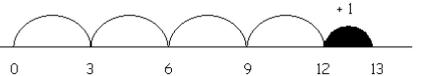| 4	Children in Year 4 should:
	• **Recall multiplication and division facts for multiplication tables up to 12 x 12**
	• Explore division situations that give rise to remainders
	• **Use place value, known and derived facts to multiply and divide mentally (e.g. 600 ÷3 = 200 can be derived from 2 x 3 = 6), including multiplying by 0 and 1; dividing by 1; multiplying together three numbers.**
	• **Use knowledge of number facts and laws of arithmetic: commutative, associative and distributive to solve mental and written calculations.**
	• **Recognize and use factor pairs**
	• Use arrays and models such as the grid method or place value counters to develop understanding of the formal methods of short multiplication and division
	• **Multiply two digit and three digit numbers by a one digit number using formal written layout of short multiplication**
	• **Use the formal written method of short division with exact answers.**
	• **Solve one and two step problems in contexts involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and correspondence problems such as when n objects are connected to m objects.**
5	Children in Year 5 should:
	• **Apply all multiplication tables and related division facts frequently, commit them to memory and use them confidently to multiply and divide numbers mentally to make larger calculations.**
	• **Develop understanding and use of factors, multiples, factor pairs, common factors and multiples, primes, prime factors, non primes (composite numbers), squares and cubes (including notation for these). Establish if a number up 100 is prime and recall primes to 19.**
	• **Multiply numbers up to 4 digits by a one or two digit number using a formal written method, including long multiplication for two-digit numbers**
	• **Divide numbers up to 4 digits by a one digit number using the formal written method of short division and interpret remainders appropriately for the context, including as fractions, decimals or by rounding.**
	• **Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000**
	• Use an expanded / informal method if they are not ready for the formal methods and be supported towards an understanding of the compact method using e.g. the grid method or place value counters
	• **Use multiplication and division facts to solve problems involving scaling by simple fractions and problems involving simple rates**
	• Learn how to record the *method* they used when working with a calculator.
6	Children in Year 6 should:
	• **Perform mental calculation, including with mixed operations and large numbers. Identify common factors, common multiples and prime numbers.**
	• **Multiply numbers up to four digits by a two digit whole number using the formal written method of long multiplication**
	• **Divide numbers up to four digits by a two digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.**
	• **Divide numbers up to four digits by a two digit whole number using the formal written method of short division where appropriate, and interpret remainders according to the context.**

- **Use knowledge of the order of operations to carry out calculations involving the four operations.**
- **Solve problems in context using all four number operations and determine, in the context of the problem, an appropriate degree of accuracy**
- They should be given opportunities to identify the most appropriate tool for calculations ie mental method, mental with recording, standard method or calculator and explain why.
- Use compact formal methods if they can do so efficiently and with understanding. Those who are not able to use a standard method should use an expanded method and work towards an understanding of a compact method.
- Learn how to record the *method* they used when working with a calculator.

Year 1	
Multiplication	**Division**

Pictures/marks

There are 3 sweets in one bag.
How many sweets are there in 5 bags?

Arrays

2 groups of 4 or 4 groups of two
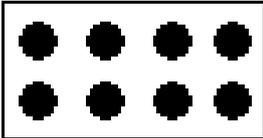
Jumps forward on a number line
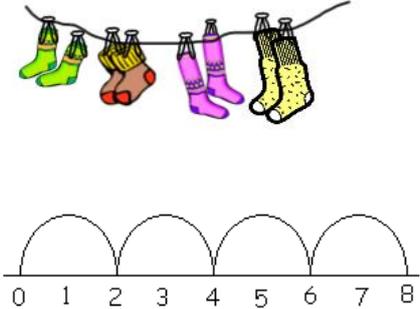
Pictures/marks

12 children get into teams of 4 to play a game. How many teams are there?

Arrays

Put into groups of two

Share between two

Jumps backwards on a number line

Year 2

Multiplication	Division
Pupils should work towards fluent recall of 2, 5 and 10 multiplication tables and derivation of related division facts and use these to calculate and solve problems. They may be supported by the type of activities shown below and regular practice of tables facts. They could explore other tables in these ways.	

Pictures/marks There are 4 apples in one box. How many apples in 6 boxes?	**Pictures/marks** 4 eggs fit in a box. How many boxes would you need to pack 20 eggs?
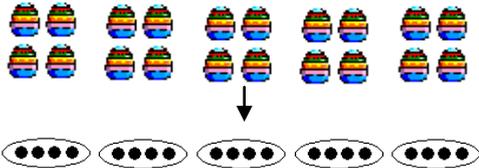
Other Jottings Arrays 2 x 4 or 4 x 2 Repeated addition $2 \times 4 = 2 + 2 + 2 + 2$ or $4 \times 2 = 4 + 4$	**Other Jottings** Arrays $8 \div 2$ or Sharing $10 \div 2$ Grouping $10 \div 2$
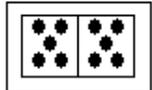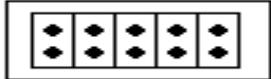
Number Lines (numbered then empty) 2 x 4 4 x 2 Recording by − drawing jumps on prepared line − constructing own lines	**Number Lines** (numbered then empty) $8 \div 2$ $8 \div 4$ Recording by - drawing jumps on prepared lines - constructing own lines
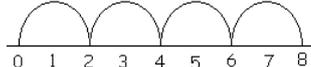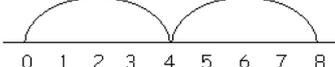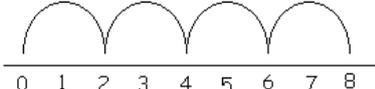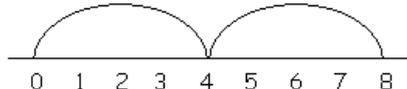
Signs and symbols $\mathscr{D}$ x 2 = $\mathscr{D}$ $\mathscr{D}$ = 2 x 6 6 x $\mathscr{D}$ = 12 12 = $\mathscr{D}$ x 6 $\mathscr{D}$ x 2 = 12 12 = $\mathscr{D}$ x $\mathscr{D}$	**Signs and Symbols** 12 $\div$ 2 = $\mathscr{D}$ $\mathscr{D}$ = 12 $\div$ 2 12 $\div$ $\mathscr{D}$ = 6 6 = $\mathscr{D}$ $\div$ 2 $\mathscr{D}$ $\div$ 2 = 6 6 = 12 $\div$ $\mathscr{D}$ $\mathscr{D}$ $\div$ $\mathscr{D}$ = 6 6 = $\mathscr{D}$ $\div$ $\mathscr{D}$ Extend to 15 – 10 = 10 $\div$ $\mathscr{D}$

Doubling by partitioning $16 \times 2 = 32$ $10 \times 2 = 20$ $6 \times 2 = 12$	**Halving by partitioning** $16 = 10 + 6$ $5 + 3 = 8$

Year 3

Multiplication	Division
The focus should be on **recall and use of the multiplication and division facts for the 3, 4 and 8 multiplication tables. They connect the 2, 4 and 8 multiplication tables through doubling.** They may use activities and recording shown below to develop understanding and promote fluency.	

Arrays

Arrays
3 x 6 or 6 x 3

Repeated addition

$3 \times 6 = 3 + 3 + 3 + 3 + 3 + 3$

or   6 x 3 = 6 + 6 + 6

Arrays

Arrays 18 ÷ 3   or

Sharing 24 ÷ 4   Grouping 24 ÷ 4

Number Lines

3 x 6

6 x 3

Number Lines

18 ÷ 3

18 ÷ 6

Remainders
13 ÷ 3

Write and calculate mathematical statements for multiplication and division.

6 x 8 = ✎ ✎ = 4 x 6 6 x ✎ = 30 30 = ✎ x 6 ✎ x 4 = 28 16 = 2 x ✎ ✎ x ✎ = 24 12 = ✎ x ✎	30 ÷ 5 = ✎ ✎ = 24 ÷ 6 12 ÷ ✎ = 6 6 = ✎ ÷ 5 ✎ ÷ 4 = 7 8 = 40 ÷ ✎ ✎ ÷ ✎ = 3 4 = ✎ ÷ ✎

Calculate two digit numbers times one digit numbers using mental and progressing to formal written methods. This could be done by developing the understanding of arrays to introduce the grid method which can then be related to the formal method.

43 x3 =

	40	3
3	**120**	**9**

Calculate two digit numbers divided by one digit numbers using mental methods.

e.g. Use multiples of the divisor and partitioning

56 ÷ 4   = (40+ 16) ÷ 4
= 10 + 4
= 14

and progression to formal written method of short division supporting understanding through the use of practical materials

43x 3   43
                x 3
 3 x 3   = 9
 40x 3 = 120
                129
	Leading to
	                43
	                x 3
	                129

14
$4\overline{)56}$

7

Year 4

Multiplication	Division
Pupils should recall multiplication and division facts for multiplication tables up to 12 x 12. They may use activities and recording shown below to develop understanding and promote fluency.	

		Division
Develop the use of arrays to aid understanding of the commutative laws	and the distributive law	Use arrays to develop ideas of remainders $34 \div 8$

$8 \times 4 = 32$ or $4 \times 8 = 32$

$8 \times 4 = (3 \times 4) + (5 \times 4)$

Use number lines

$4 \times 7 = $

0 4 8 12 16 20 24 28

7×4 =

0 7 14 21 28

Number lines (including remainders)

$23 \div 7$

or

+2

0 7 14 21 23

$23 \div 7$

-7   -7   -7

2   9   16   23

Record multiplication and division facts

3 x 7 = ✎	✎ = 7 x 3	21 ÷ 7 = ✎	✎ = 21 ÷ 7
3 x ✎ = 21	21 = ✎ x 3	21 ÷ ✎ = 3	3 = ✎ ÷ 7
✎ x 7 = 21	21 = 7 x ✎	✎ ÷ 7 = 3	3 = 21 ÷ ✎
✎ x ✎ = 21	21 = ✎ x ✎	✎ ÷ ✎ = 3	3 = ✎ ÷ ✎

Use place value, known and derived facts to multiply and divide mentally.
$30 \times 6 = 3 \times 6 \times 10 = 18 \times 10 = 180$

Multiply two and three digit numbers by a one digit number	Division
Multiply two and three digit numbers by a one digit number using the formal written layout They may use informal or expanded methods to support understanding of the formal written method. Grid method **23 x 7 =** × 20 3 7 [140 21] = 161	Continue to calculate two digit numbers divided by one digit numbers using mental methods. e.g. Use multiples of the divisor and partitioning $72 \div 5 = (50 + 22) \div 5$ $= 10 + 4 \text{ remainder } 2$ $= 14 \text{ remainder } 2$

| Expanded method

 23 x 7 =   23
   x 7
 3 x 7   21
 20 x 7   140
   161 | Develop understanding of and practice **the formal written method of short division with exact answers,** supported by practical materials if required

 1   4
 2
 7 ⎸ 9   8

 Answer: 14 |

Leading to **the formal written method to multiply two and three digit numbers by a one digit number.**

23
x 7
161
2

3   4   2
×   7
2   3   9   4
2   1

Answer: 2394

Year 5

Continue to practice and apply all tables facts and commit to memory.	
Multiplication	**Division**

Multiplication	Division
Signs & Symbols Use signs and symbols to complete questions using known and derived facts eg $9 \times 7 = \varnothing$ $\varnothing = 60 \times 2$ $\varnothing \times 7 = 63$ $120 = \varnothing \times 2$ $9 \times \varnothing = 63$ $120 = 60 \times \varnothing$ $\varnothing \times \varnothing = 63$ $120 = \varnothing \times \varnothing$	**Signs & Symbols** Use signs and symbols to complete questions using known and derived facts eg $56 \div 7 = \varnothing$ $1600 \div 2 = \varnothing$ $\varnothing \div 7 = 8$ $\varnothing \div 2 = 800$ $56 \div \varnothing = 8$ $1600 \div \varnothing = 800$ $\varnothing \div \varnothing = 8$ $\varnothing \div \varnothing = 800$

Number Sentences
Use number sentences to show mental strategy used.
eg (i) $36 \times 50 = 1800$
         $36 \times 100 = 3600$
         $3600 \div 2 = 1800$
 	
   (ii) $15 \times 6 = 90$
         $15 \times 3 = 45$
         $45 \times 2 = 90$
	Number Sentences
	Use number sentences to show mental strategy used.
	e.g.(i)
	$198 \div 6 =$
	$(180 + 18) \div 6 =$
	$30 + 3 = 33$
	e.g.(ii)
	$345 \div 15 =$
	$(300 + 45) \div 15 =$
	$20 + 3 = 23$

Most children • **Multiply numbers up to 4 digits by a one or two digit number using a formal written method, including long multiplication for two-digit numbers**	Most children **Divide number up to 4 digits by a one-digit number, using the formal written method of short division and interpret remainders appropriately for the context**

Short multiplication	**Long multiplication for two digit numbers**	**Short division of number up to 4 digits**
342×7 becomes $\begin{array}{r} 3\ 4\ 2 \\ \times\ \ \ \ \ 7 \\ \hline 2\ 3\ 9\ 4 \\ \ \ 2\ 1 \end{array}$ Answer: 2394	24×16 becomes $\begin{array}{r} 2 \\ 2\ 4 \\ \times\ \ 1\ 6 \\ \hline 2\ 4\ 0 \\ 1\ 4\ 4 \\ \hline 3\ 8\ 4 \end{array}$ Answer: 384	$432 \div 5$ becomes $496 \div 11$ becomes $\begin{array}{r} 8\ \ 6\ r\,2 \\ 5\ \overline{)4\ 3\ 2^{\,3}} \end{array}$ $\begin{array}{r} 4\ \ 5\ r\,1 \\ 1\ 1\ \overline{)4\ 9\ 6^{\,5}} \end{array}$ Answer: 86 remainder 2 Answer: $45\frac{1}{11}$

Multiplication	Division
Use informal written methods for those not ready for the formal method or to develop understanding **Grid Method** 346×9 	$\times$	300	40	6	
---	---	---	---	---
9	2700	360	54	= 3114 | 72×38 is approximately $70 \times 40 = 2800$ 72×38 | $\times$	70	2	
---	---	---	---
30	2100	60	2160
8	560	16	$+\ 576$
			2732		
Expanded methods $\begin{array}{rr} & 346 \\ & \times\ \ \ 9 \\ \hline 6 \times 9 & 54 \\ 40 \times 9 & 360 \\ 300 \times 9 & 2700 \\ \hline & 3114 \end{array}$	

Year 6

Multiplication	Division
Continue to practice and complete multiplications and divisions	

8 x 9 = ✎ 370 x 2 = ✎ ✎ x 9 = 72 176 x 2 = ✎ 8 x ✎ = 72 ✎ x 2 = 3.9 ✎ x ✎ = 72	72 ÷ 9 = ✎ 1750 ÷ ✎ = 875 72 ÷ ✎ = 8 570 ÷ 2 = ✎ ✎ ÷ 9 = 8 ✎ ÷ 2 = 0.87 ✎ ÷ ✎ = 8

Use number sentences to show mental strategy used.
eg   38 x 25 =
        38 x 100 = 3800
        3800 ÷ 4 = 950
eg   35 x 18 = 630
        35 x 6 = 210
        210 x 3 = 630
	Use number sentences to show mental strategy used.
	e.g.(i)
	198 ÷ 6 =
	(180 + 18) ÷ 6 =
	30 + 3 = 33
	e.g.(ii)
	345 ÷ 15 =
	(300 + 45) ÷ 15 =
	20 + 3 =23

Most pupils multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication	**Most pupils:** • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. • divide numbers up to 4 digits by a two-digit whole number using the formal written method of short division where appropriate and interpret remainders according to context. •
Informal Written Methods Continue to develop written method and use expanded methods for children who are not ready for a compact method to develop understanding Grid Method 	
| **Standard written methods**<br><br>Short multiplication: ThHTU x U<br><br>```
              4346
           x     8
    6 x 8      48
    40 x 8    320
   300 x 8   2400
  4000 x 8  32000
             34768
``` | |