

## educationGateshead

### Guidance document: Developing calculation across Key Stages 1 and 2

#### Introduction

This document recommends an approach for developing progression in the conceptual and procedural aspects of calculation across Key Stages 1 and 2. It takes into account the mathematics programmes of study and non statutory guidance for the National Curriculum 2014.

The document covers:

- Calculations that can be done wholly or partially by mental methods, based on fluency with number facts and understanding of place value and number operations and sometimes using horizontally presented number sentences or empty number lines to show steps in thinking.
- The use of expanded or informal written methods to support understanding of compact, formal written methods and
- Developing fluency in the use of formal written methods by the end of key stage 2.
- Appropriate use of a calculator.

Children should work towards being able to use, by the end of Key Stage 2:

- A range of strategies for mental calculations *appropriate to the numbers involved*.
- One formal written method (for each number operation) for calculations that cannot be done mentally.
- A calculator for calculations where this is the most appropriate choice.

Progression in mental calculation skills can be supported by:

- The ability to quickly recall a range of number facts and an understanding of how to use them to derive other related facts.
- Understanding how numbers and calculations can be represented by materials and images such as arrays, ten frames, Numicon shapes.
- An understanding of the number system (order and relative position of numbers, place value, etc), the four number operations and the laws of arithmetic associated with them
- Understanding of how symbols are used to record calculations especially the equals sign. Care should be taken that the equals sign is used correctly
  - Eg  $42 + 35 =$   might be calculated by partitioning the second number to add the tens followed by the units. This could be recorded as:  
 $42 + 30 = 72$   
 $72 + 5 = 77$   
*But not as  $42 + 30 = 72 + 5 = 77$*   
as this involves an incorrect use of the first equals sign.
- An understanding of how calculations can be represented on empty number lines. They will need to work with numbered tracks and lines first before they are confident to rely on empty lines alone. To make good use of empty lines children need to be able to:
  - Move forward and back confidently on the number line.
  - Make jumps of different sizes.
  - Recognize landmark numbers such as multiples of 10

- **Know** and use number complements to 10 and how these relate to multiples of 10.
- Partition and recombine numbers in appropriate ways eg  $7 + 5$  as  $7 + 3 + 2$ , or  $28 + 9$  as  $28 + 10 - 1$ .

Teachers should **demonstrate** the use of number sentences and number lines to **model** steps in calculations. Children should be encouraged to record the steps in their mental calculations some of the time. Recording is useful when explaining methods to others and to show which strategy has been used. It is not necessary to always record, especially for those children who have efficient mental methods. Teachers should use their judgement about when to require recording.

Progression to fluency with a formal written method for each number operation can be made by

- The appropriate use of informal or expanded written methods that build on mental methods and which continue to highlight understanding of the number system and number operations.
- Linking of these expanded methods to the formal written method when it is first introduced to highlight steps that may be concealed, and hence not understood, in the procedural execution of the formal written method.
- Appropriate levels of practice of formal written methods to develop fluency.

Children should continue to develop their mental calculation skills with larger numbers once written methods are introduced and should be given opportunities to identify which calculations might be done mentally, with reference to the **nature** rather than **magnitude** of the numbers involved. They should use mental calculation skills to estimate the likely magnitude of the answer when performing a calculation using a formal written method and hence identify answers that are unreasonable and indicate errors in execution of the method.

Teachers need to judge when children are ready to move from mental to written calculations. The following lists offer some guidance.

### **Addition and subtraction**

#### **Can pupils:**

- recall addition and subtraction facts to 20?
- understand place value and partition numbers?
- add three single digit numbers mentally?
- add or subtract any pair of two digit numbers mentally?
- explain their mental strategies orally and record them using horizontal number sentences or an empty number line?

### **Multiplication and division**

#### **Can pupils:**

- quickly recall multiplication and division facts for 2, 3, 4, 5 and 10 times tables?
- understand what happens when a number is multiplied by 0 or 1?
- understand 0 as a place holder?
- multiply two- and three-digit numbers mentally by 10 and 100?
- demonstrate understanding of the commutative, distributive and associative laws (though not necessarily know the names)?
- double and halve two-digit numbers mentally?
- explain mental strategies orally and with recording?

This document considers addition and subtraction together followed by multiplication and division. **Links between number operations should be emphasised regularly.** A year-by-year approach has been taken in line with the format of the National Curriculum 2014 programmes of study but teachers should have regard to other year group expectations when planning for different abilities.

### Addition and subtraction

The first table below gives an overview of the calculation expectations for each year group. Statements highlighted in **bold** can be matched to the National Curriculum 2014 programmes of study or non statutory guidance. Other items are suggested approaches for schools to follow to support children’s understanding of calculation methods.

The second table in this section sets of how children’s recording of calculations might look depending on the mental strategy or written method being used.

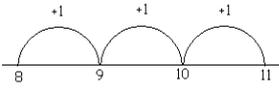
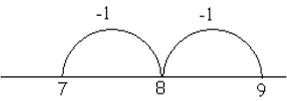
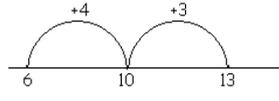
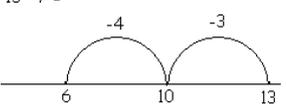
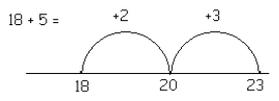
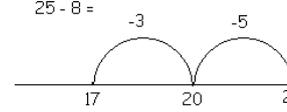
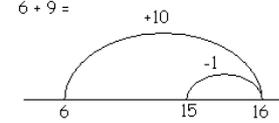
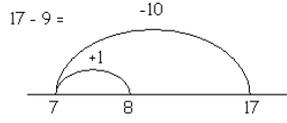
Year	Addition and subtraction
1	<p>Children in Year 1 should:</p> <ul style="list-style-type: none"> <li>• <b>Use concrete objects and pictorial representations, including number lines, to support their solution of addition and subtraction problems.</b></li> <li>• <b>Represent and use number bonds and related subtraction facts within 20, memorizing and reasoning with these bonds.</b></li> <li>• <b>Add and subtract one-digit and two-digit numbers to 20, including zero (and realize the effect of adding or subtracting zero to establish the relationship between these operations)</b></li> <li>• <b>Read, write and interpret mathematical statements involving addition (+), subtraction (-) and (=) signs in a range of formats e.g. <math>\Delta + 5 = 12</math> or <math>7 = \diamond - 9</math></b></li> </ul>
2	<p>Children in Year 2 should:</p> <ul style="list-style-type: none"> <li>• <b>Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 e.g. use <math>3 + 7 = 10</math> to derive <math>30 + 70 = 100</math></b></li> <li>• <b>Use concrete objects and pictorial representations to support their solution of addition and subtraction problems and to add and subtract mentally including TU+/-U, TU+/- T, TU +/- TU, U + U + U.</b></li> <li>• <b>Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot. Use the associative law of addition to show for example that <math>5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5</math></b></li> <li>• <b>Recognize and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems</b></li> <li>• Record mental additions and subtractions using horizontal number sentences and/or empty number lines to show and explain the steps in their calculations.</li> <li>• <b>Recording in columns supports place value and prepares for formal methods.</b></li> </ul>
3	<p>Children in Year 3 should:</p> <ul style="list-style-type: none"> <li>• <b>Add and subtract numbers mentally including HTU +/- U, HTU+/- T, HTU+/- H</b></li> <li>• Use horizontal number sentences and empty number lines sometimes to support explanation of their mental calculation methods.</li> <li>• <b>Solve varied addition and subtraction problems including missing number problems using number facts and place value.</b></li> <li>• Develop their understanding of written methods; working from expanded to <b>using (compact) formal written methods of columnar addition and subtraction with numbers of up to three digits.</b> Particular attention should be paid to the language used when modeling these methods. The value of digits should be retained according to place value and use of practical materials /representations may aid understanding</li> <li>• <b>Estimate the answer to a calculation and check using inverse operations.</b></li> </ul>

4	<p>Children in Year 4 should:</p> <ul style="list-style-type: none"> <li>• <b>Continue to add and subtract numbers</b> with up to four digits <b>mentally</b> where the nature of the numbers makes this appropriate. They may use horizontal number sentences or empty number lines to support an explanation of the steps in their calculation. They should be given opportunities to identify calculations which are appropriate for a mental method and explain why.</li> <li>• <b>Add and subtract numbers with up to four digits using the formal written methods of columnar addition and subtraction where appropriate.</b> Their understanding of the procedures involved may be supported by the use of expanded written methods and practical materials if required.</li> <li>• <b>Estimate and use inverse operations to check answers to a calculation.</b></li> </ul>
5	<p>Children in Year 5 should:</p> <ul style="list-style-type: none"> <li>• <b>Add and subtract numbers mentally with increasingly large numbers e.g. <math>12,462 - 2300 = 10,162</math>.</b> Use horizontal number sentences and empty number lines sometimes to support explanation of their methods. They should be given opportunities to identify calculations which are appropriate for a mental method and explain why.</li> <li>• <b>Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction).</b> Particular attention should be paid to the language used when modelling these methods. The value of digits should be retained according to their place value. Understanding of the procedures involved may be supported by the use of expanded written methods and practical materials if required.</li> <li>• <b>Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy</b></li> <li>• <b>Solve addition and subtraction multi-step problems in context, deciding which operations to use and why.</b></li> <li>• Learn how to record the method they used when working with a calculator.</li> </ul>
6	<p>Children in Year 6 should:</p> <ul style="list-style-type: none"> <li>• <b>Perform mental calculations including with mixed operations and large numbers (and decimals).</b> Use horizontal number sentences and empty number lines sometimes to support explanation of their methods. They should be given opportunities to identify the most appropriate tool for calculations ie mental method, mental with recording, formal written method or calculator and explain why.</li> <li>• <b>Practise addition and subtraction for (appropriate) larger numbers and decimals using the formal written methods of columnar addition and subtraction.</b> Those who are not able to use the compact formal method may use an expanded method and work towards an understanding of the formal written method. Particular attention should be paid to the language used when modelling these methods. The value of digits should be retained according to their place value. Materials / representations may support understanding.</li> <li>• <b>Use estimation to check answers to calculations and determine, in the context of the problem, an appropriate degree of accuracy.</b></li> <li>• <b>Round answers to a specified degree of accuracy.</b></li> <li>• <b>Use knowledge of the order of operations, and use of brackets, to carry out calculations involving the four operations.</b></li> <li>• <b>Solve addition and subtraction multi-step problems in contexts, deciding which operations to use and why.</b></li> <li>• Learn how to record the method they used when working with a calculator.</li> </ul>

**YEAR 1**

**NOTE: Pupils should memorise and reason with number bonds to 10 and 20.**  
Use of structured materials such as ten frames may support this and reduce dependence on count by ones strategies.

**Pupils should become familiar with the terms: *put together, add, altogether, total, take away, distance between, difference between, more than and less than* so they develop the concept of addition and subtraction and can use these operations flexibly.**

Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
Counting on/back	$8 + 3 = 11$		$9 - 2 = 7$	
Reordering: Count on from larger number	$3 + 8 =$  rewrite as $8 + 3 = 11$	Reorder before using number line as above		
Find pairs that total 10	$3 + 4 + 7 =$ $3 + 7 + 4 =$ $10 + 4 = 14$			
Partition into 5 and a bit	$5 + 8 =$  $5 + 5 + 3 =$ $10 + 3 = 13$			
	$7 + 8 =$  $5 + 2 + 5 + 3 =$ $5 + 5 + 2 + 3 =$ $10 + 5 = 15$			
Use near doubles	$5 + 6 =$  $5 + 5 + 1 =$  $10 + 1 = 11$			
Begin to bridge through 10	$6 + 7 =$  $6 + 4 + 3 =$ $10 + 3 = 13$	$6 + 7 =$ 	$13 - 7 =$  $13 - 3 - 4 =$ $10 - 4 = 6$	$13 - 7 =$ 
	$18 + 5 =$  $18 + 2 + 3 =$ $20 + 3 = 23$	$18 + 5 =$ 	$25 - 8 =$  $25 - 5 - 3 =$ $20 - 3 = 17$	$25 - 8 =$ 
Add or subtract 9	$6 + 9 =$  $6 + 10 - 1 =$ $16 - 1 = 15$	$6 + 9 =$ 	$17 - 9 =$  $17 - 10 + 1 =$ $=$ $7 + 1 = 8$	$17 - 9 =$ 

**YEAR 2**

**Establish the use of efficient, non counting based, strategies using bonds to 20, place value etc.**

Use of representations and materials such as ten frames and base ten materials may support understanding.

Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
Counting on/back in ones and tens. <b>Move children to using known bonds to reduce reliance on this.</b>	$34 + 3 =$ $23 + 20 =$	 	$27 - 4 =$ $45 - 20 =$	 
Count up to find a small difference			$82 - 79 =$	
Reordering	$5 + 7 + 5 =$ $5 + 5 + 7 =$			
Use near doubles	$6 + 7 =$ $6 + 6 + 1 =$ $40 + 39 =$ $40 + 40 - 1 =$			
Bridge through multiples of 10	$25 + 7 =$ $25 + 5 + 2 =$		$45 - 8 =$ $45 - 5 - 3 =$	
Partitioning using multiples of 10. <i>Partition both numbers or just the second.</i>	$25 + 14 =$ $20 + 10 = 30$ $5 + 4 = 9$ or $25 + 10 = 35$ $35 + 4 = 39$		$46 - 23 =$ $40 - 20 = 20$ $6 - 3 = 3$ or $46 - 20 = 26$ $26 - 3 = 23$ NB In cases such as $43 - 26 =$ $30 \quad 40 - 20 =$ $13 \quad 3 - 6 =$	
Compensating to add/subtract numbers close to a multiple of 10	$24 + 19 =$ $24 + 20 - 1 =$ $58 + 21 =$ $58 + 20 + 1 =$	 	$70 - 11 =$ $70 - 10 - 1 =$ $53 - 19 =$ $53 - 20 + 1 =$	 

YEAR 3				
Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p><b>Children should calculate mentally with up to three digit numbers when nature of numbers makes this appropriate.</b> They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but may be able to work without jottings e.g. <math>433 + 200</math>, <math>385 - 40</math>, <math>501 - 4</math>. They may use the following strategies and recording.</p>				
Add/subtract ones, tens and hundreds using number bonds and place value to find most efficient steps.	$90 + 40 =$		$110 - 30 =$	
Count up to find a difference			$504 - 498 =$	
Bridge through multiples of 10	$49 + 7 =$ $49 + 1 + 6 =$		$62 - 7 =$ $62 - 2 - 5 =$	
Compensating to add/subtract numbers close to a multiple of 10	$543 + 29 =$ $543 + 30 - 1 =$		$273 - 29 =$ $273 - 30 + 1 =$	
Partitioning using multiples of 10 <i>Partition both numbers or just the second</i>	$86 + 57 =$ $80 + 50 = 130$ $6 + 7 = 13$ or $86 + 50 = 136$ $136 + 7 = 143$		$96 - 24 =$ $90 - 20 = 70$ $6 - 4 = 2$ or $96 - 20 = 76$ $76 - 4 = 72$	

### Introducing the formal written method of columnar addition and subtraction

	Addition	Subtraction
Partitioning both numbers using multiples of 10 and using the expanded method may help children move to the formal written method with understanding.	$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \\ \hline 80 \\ \hline 91 \end{array}$	$\begin{array}{r} 87 = 80 + 7 \\ - 53 \quad \underline{50 + 3} \\ \quad 30 + 4 \\ \\ 83 = 80 + 3 \\ - 57 \quad \underline{50 + 7} \\ \quad 20 + 6 \end{array}$
Introduce the <b>formal method</b> with or without regrouping as appropriate for pupils	$\begin{array}{r} 234 \\ + 145 \\ \hline 379 \end{array}$ $\begin{array}{r} 234 \\ + 178 \\ \hline 412 \\ 11 \end{array}$	$\begin{array}{r} 285 \\ - 123 \\ \hline 162 \end{array}$ $\begin{array}{r} 71 \\ 285 \\ - 127 \\ \hline 158 \end{array}$

YEAR 4				
Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p>Children should calculate mentally with up to four digit numbers when nature of numbers makes this appropriate. They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but may be able to work without jottings e.g. <math>1433 + 200</math>, <math>985 - 420</math>, <math>1510 - 40</math>. They may use the following strategies.</p>				
Count up to find a small difference			$403 - 386 =$ $4008 - 3993 =$	
Bridge through multiples of 10	$357 + 7 =$ $357 + 3 + 4 =$		$905 - 7 =$ $905 - 5 - 2 =$	
Compensating to add/subtract numbers close to a multiple of 10	$74 + 58 =$ $74 + 60 - 2 =$		$283 - 71 =$ $283 - 70 - 1 =$	
Partitioning using multiples of 10 <i>Partition both numbers or just the second.</i>	$88 + 76 =$ $80 + 70 = 150$ $8 + 6 = 14$ or $88 + 70 = 158$ $158 + 6 = 164$		$98 - 43 =$ $90 - 40 = 50$ $8 - 3 = 5$ or $98 - 40 = 58$ $58 - 3 = 55$	

### Developing the formal written method of columnar addition and subtraction

	Addition	Subtraction
<p><b>Most children should add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction.</b></p>		
For children who are not ready for these formal methods use: <ul style="list-style-type: none"> <li>the informal expanded method for addition, adding the least significant digits first and</li> <li>the expanded decomposition method for subtraction</li> </ul> Move from these when ready to the <b>formal written method</b> supporting children to understand regroupings involved. Place value materials and representations may support children to understand the written procedure if required	$\begin{array}{r} 358 \\ + 73 \\ \hline 11 \\ 120 \\ \underline{300} \\ 431 \end{array}$	$\begin{array}{r} 40 + 14 \\ 754 = 700 + 50 + 4 \\ - 36 \\ \hline 30 + 6 \\ 700 + 10 + 8 \\ \hline 718 \end{array}$

YEAR 5				
Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p><b>Children should calculate mentally with large numbers when nature of numbers makes this appropriate e.g. <math>12,462 - 2,300 = 10,162</math>.</b> They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but may be able to work without jottings. They may use the following strategies.</p>				
Count up to find a small difference			$705 - 287 =$ $8006 - 2993 =$	
Bridge through whole numbers for decimals	$3.8 + 2.6 =$ $3.8 + 0.2 + 2.4 =$		$7.5 - 0.8 =$ $7.5 - 0.5 - 0.3 =$	
Compensating to add/subtract numbers close to a multiple of 10	$346 + 59 =$ $346 + 60 - 1 =$ $406 - 1 = 405$  $478 + 71 =$ $478 + 70 + 1 = 549$	 	$425 - 58 =$ $425 - 60 + 2 =$ $365 + 2 = 367$  $583 - 71 =$ $583 - 70 - 1 =$	 
Partition using multiples of 10. <i>Partition both numbers or just the second.</i>	$324 + 58 =$ $320 + 50 = 370$ $4 + 8 = 12$ or $324 + 50 = 374$ $374 + 8 = 382$		$428 - 43 =$ $428 - 40 - 3 =$	

	Addition	Subtraction
<p><b>Most children use the formal written methods of columnar addition and subtraction with appropriate whole numbers of more than 4 digits</b></p>		
<p><b>Most children use the formal written methods.</b></p>	$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \small{11} \end{array}$	$\begin{array}{r} 754 \\ - 286 \\ \hline 468 \end{array}$
<p>Use the expanded methods to support children who are not ready for the compact written method to understand and move towards use of the formal methods. Place value materials and representations may support children to understand the written procedure if required.</p>	$\begin{array}{r} 587 \\ +475 \\ \hline 12 \\ 150 \\ \hline 900 \\ \hline 1062 \end{array}$	$\begin{array}{r} 600 + 140 \\ \quad 40 + 14 \\ 754 = 700 + 50 + 4 \\ -286 \quad \underline{200 + 80 + 6} \\ \quad 400 + 60 + 8 \end{array}$

YEAR 6				
Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p><b>Children should continue to calculate mentally with large numbers and decimals when nature of numbers makes this appropriate e.g. <math>12,462 - 2,300 = 10,162</math>.</b> They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but will often be able to work without jottings. They may use the following strategies.</p>				
Count up to find a small difference			$8004 - 2785 =$	
Bridge through whole numbers for decimals	$3.8 + 2.6 =$ $3.8 + 0.2 + 2.4 =$		$7.5 - 0.8 =$ $7.5 - 0.5 - 0.3 =$	
Compensating to add/subtract numbers close to a multiple of 10 and whole numbers when working with decimals	$7.5 + 0.9 =$ $7.5 + 1.0 - 0.1 =$		$19.3 - 2.9 =$ $19.3 - 3.0 + 0.1 =$	
Partition using multiples of 10. <i>Partition both numbers or just the second.</i>	$540 + 280 =$ $540 + 200 + 80 =$		$276 - 153 =$ $276 - 100 - 50 - 3 =$	

	Addition	Subtraction
<p><b>Most children use the formal written methods of columnar addition and subtraction.</b></p>		
<p><b>Most children use the formal written methods.</b>            Extend practice to numbers with any number of digits and to two or three decimal places.</p> <p>Use expanded method for those who are not ready for the formal written method and support understanding through the use of place value materials and smaller numbers as appropriate.</p>	$\begin{array}{r} 7648 \\ +1486 \\ \hline 9134 \\ \hline 111 \end{array}$	$\begin{array}{r} 5 \ 13 \ 1 \\ 6 \ 4 \ 6 \ 7 \\ - 2 \ 6 \ 8 \ 4 \\ \hline 3 \ 7 \ 8 \ 3 \end{array}$
	$\begin{array}{r} 7648 \\ +1486 \\ \hline 14 \\ 120 \\ 1000 \\ \hline 8000 \\ \hline 9134 \end{array}$	$\begin{array}{r} 600 + 140 \\ 40 + 14 \\ 754 = 700 + 50 + 4 \\ -286 \quad \underline{200 + 80 + 6} \\ 400 + 60 + 8 \end{array}$